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Reg. No. : ....

Code No. : 30339 E **Sub. Code : JMMA 61**

B.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2020.

Sixth Semester

Mathematics – Main

LINEAR ALGEBRA

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Under which scalar multiplication defined below, $R \times R$ is not a vector space over R?

   (a) $\alpha(x, y) = (\alpha x, \alpha^2 y)$

   (b) $\alpha(a, b) = (0, 0)$

   (c) $\alpha(a, b) = (\alpha a, 0)$

   (d) All the above

2. The union of two subspaces A and B is a vector space if

(a) $A \cap B = \{\ \}$

(b) $B \subset A'$

(c) $A \subseteq B$ or $B \subseteq A$

(d) $B = A'$

3. If $L(S) = S$, then for the vector space V, S is a/an ————

(a) empty set (b) equal set

(c) equal space (d) subspace

4. The vector space of all polynomials of degree $\leq n$ on $R[x]$ over R has dimension ————

(a) $n - 1$ (b) n

(c) $n + 1$ (d) 1

5. For the linear transformations T_1 and T_2 if rank $(T_2 T_1) = rank\ T_2$, then ————

(a) T_1 is $1 - 1$

(b) T_2 is $1 - 1$

(c) Both (a) and (b)

(d) Neither (a) nor (b)

6. The vector of unit length normal to the vector $(1, 3, 4)$ is

(a) $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (b) $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$

(c) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$ (d) $(1, -3, 4)$

7. The inverse of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is ———————

(a) $\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$ (b) $\frac{1}{2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$

(c) $\frac{1}{2}\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}$

8. Which of the following is true?

(a) $rank\ A > rank(A, B)$

(b) $rank\ A \geq rank\ (A, B)$

(c) $rank\ (A, B) \leq 0$

(d) $rank\ A \leq rank(A, B)$

9. If zero is an eigen value of A , then ———————

(a) $|A| = 0$ (b) $|A| \neq 0$

(c) $|A| > 0$ (d) $|A| < 0$

10. The matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ satisfies the equation ____________

(a) $A^2 - 2A + 5I = 0$

(b) $A^2 - 2A - 5I = 0$

(c) $A^2 + 2A - 5I = 0$

(d) $A^2 + 2A + 5I = 0$

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let V be a vector space over a field F. Prove that a non-empty subset W of V is a subspace if and only if $u, v \in W$ and $\alpha, \beta \in F \Rightarrow \alpha\, u + \beta v \in W$.

Or

(b) (i) $T : R^2 \to R^2$ defined by $T(a, b) = (2a - 3b,\ a + 4b)$ is a linear transformation. Prove.

(ii) Show that if $T : V \to W$ is a linear transformation, $T(V)$ is a subspace of W.

12. (a) Let V be a finite dimensional vector space over F. Then prove that any linearly independent set of vectors in V is a part of a basis.

Or

(b) Show that any two vector spaces of the same dimension over a field F are isomorphic.

13. (a) Let V and W be two finite dimensional vector spaces over F. Let $\dim V = m$ and $\dim W = n$. Show that $L(V, W)$ is a vector space of dimension mn over F.

Or

(b) State and prove Schwartz's inequality and triangle inequality.

14. (a) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & -1 \\ -2 & 1 & 3 \end{pmatrix}$.

Or

(b) Show that the system of equations is inconsistent.

$x + 2y + z = 11$; $4x + 6y + 5z = 8$;

$2x + 2y + 3z = 19$.

15. (a) Verify Cayley Hamilton theorem for $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$.

Or

(b) The product of two eigen values of the matrix $A = \begin{pmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{pmatrix}$ is –12. Find all eigen values of A.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that for a vector space V and its subspace W over F, $V/W = \{W + v / v \in V\}$ is a vector space over F.

Or

(b) State and prove fundamental theorem of homomorphism.

17. (a) Show that if $V = A \oplus B$, then $\dim V = \dim A + \dim B$.

Or

(b) Prove that any two bases of a finite dimensional vector space have same number of elements.

18. (a) Show that every finite dimensional inner product space has an orthonormal basis.

Or

(b) Let W_1 and W_2 be subspaces of a finite dimensional inner produce space. Then show that

(i) $(W_1 + W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$.

(ii) $(W_1 \cap W_2)^{\perp} = W_1^{\perp} + W_2^{\perp}$.

19. (a) Find the rank of the matrix $A = \begin{pmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 7 \end{pmatrix}$.

Or

(b) For what values of η, the equations are consistent? $x + y + z = 1$; $x + 2y + 4z = \eta$; $x + 4y + 10z = \eta^2$. And solve them.

20. (a) State and prove Cayley Hamilton theorem. Using this theorem find the inverse of $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$.

Or

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.